HONG KONG STATISTICAL SOCIETY

2016 EXAMINATIONS – SOLUTIONS

HIGHER CERTIFICATE – MODULE 7

The Society is providing these solutions to assist candidates preparing for the examinations in 2017.

The solutions are intended as learning aids and should not be seen as "model answers".

Users of the solutions should always be aware that in many cases there are valid alternative methods. Also, in the many cases where discussion is called for, there may be other valid points that could be made.

While every care has been taken with the preparation of these solutions, the Society will not be responsible for any errors or omissions.

The Society will not enter into any correspondence in respect of these solutions.

HC7 2016: Solution to Q1

(i) As C(t) is the seasonal component for quarterly data it is constant in each quarter, ie C(1)=C(5) = C(9) and similarly for C(2), C(3), C(4). 1, 1
Hence C(t) = C(t-4).
Standardising condition: C(1) + C(2) + C(3) + C(4) = 0. 1

(ii) Q2 2014 : (322+333+351)/3 = 335.3 Error = 333 - 335.3 = -2.3
Q3 2014 : (333+351+410)/3 = 364.6 Error = 351 - 364.6 = -13.6

[1 mark for method, 2 accuracy (1 each for Q2 and Q3)]

(iii) A 3-point moving average doesn't span the whole cycle so will still show seasonal variation.

(iv)To get C(1, … C(4) average the errors then subtract their mean to standardise to sum = 0. 1

	C(1)	C(2)	C(3)	C(4)	Mean
Average error	-11.335	0.11	-12.56	27.165	0.845
Corrected C(t)	-12.18	-0.735	-13.405	26.32	0

1 for C(1) - C(4)
1 for Mean
1 for Corrected

(v)* Q3 2015 has t = 15 so y(15) = 254.99 + 15×8.72 – 13.405 = 372.385.
Q4 2015 has t = 16 so y(16) = 254.99 + 16×8.72 + 26.32 = 420.83.

[1 mark for method, 2 for accuracy (again 1 each for Q3 and Q4)]

(vi) Calculate t-ratios = Estimate/Std. Error and compare with Student's t on 9df. 1

	Intercept	t	Q4
t-ratio	30.26	8.16	5.23

1

All three are significant at 0.1% so strong evidence that both t and Q4 should be retained in B's model. 1, 1

(vii) Q3 2015 : y(15) = 242.282 + 15×9.157 = 379.64
Q4 2015 : y(16) = 242.282 + 16×9.157 + 46.798 = 435.59 3 — 1
Both of B's forecasts are higher than A's. 1

* Follow through in part (v) if uncorrected seasonal estimates used :
Q3 2015 has t = 15 so y(15) = 254.99 + 15×8.72 – 12.56 = 373.23.
Q4 2015 has t = 16 so y(16) = 254.99 + 16×8.72 + 27.165 = 421.675.
(For 3 marks)

20

2. (i) Increasing, non-linear trend. 1.
No apparent seasonal pattern. 1.
/3 More volatile as level increases. 1

(ii) Aim is to transform to a stationary series.
Logs may stabilise the variance and linearise 1.
the trend. 1st differences would remove a
/2 linear trend leaving the level constant. 1

(iii) $p=2,\ d=0,\ q=0$ 1.
$y_t = 0.0278 - 0.7983\,y_{t-1} - 0.02276\,y_{t-2} + \varepsilon_t$ 2.
ε_t is assumed white noise, usually Normal.
/6 $E[\varepsilon_t]=0$, $Var[\varepsilon_t]=\sigma^2$ constant (ie $N(0,\sigma^2)$). 1,1
$Cov[\varepsilon_t, \varepsilon_{t-i}]=0$ for all $i=1,2,3,\ldots$ 1

(iv) x_t would be ARIMA(2,1,0). 1.
The constant in the model for y_t corresponds
/2 to the increasing (linear) trend in x_t. 1.

(v) The acf should be zero if the model is correct. 1.
All residual autocorrelations lie within the
dotted parallel lines so would not be judged 1.
/3 significantly different from zero. 1.

(vi) Forecast: $\hat{y}_{77}(1) = 0.0278 - 0.7983\,y_{76} - 0.02776\,y_{75}$, 2.
$\hat{x}_{77}(1)$ could be calculated as a cumulative
sum of $0.0278 - 0.7983\,y_t - 0.02776\,y_{t-1}$. 1.
Then $\exp(\cdot)$ to transform back to the
/4 original scale. 1.

20

3 (i) $Q_L(0,t) = \sum_i p_{oi} q_{ti} / \sum_i p_{oi} q_{oi}$ 1.

3 Denominator: $\sum p_0 q_0$ is total revenue/value at time 0, 1.
Numerator: $\sum p_0 q_t$ is notional revenue if q_t had been sold at time zero prices, p_0. 1.

1 (ii) $P_P(0,t) = \sum_i q_{ti} p_{ti} / \sum_i q_{ti} p_{oi}$ 1.

(iii) $\sum_i p_{oi} q_{ti} = \frac{\sum_i p_{oi} q_{ti}}{\sum_i p_{ti} q_{ti}} \times \sum_i p_{ti} q_{ti}$ 1

$= \sum_i p_{ti} q_{ti} \times \left[\frac{\sum_i q_{ti} p_{oi}}{\sum_i q_{ti} p_{ti}} \right]^{-1}$ 1.

$= \sum_i p_{ti} q_{ti} / P_P(0,t)$ 1.

$\therefore Q_L(0,t) = \frac{\sum_i p_{ti} q_{ti}}{\sum_i p_{oi} q_{oi}} / P_P(0,t)$ 1.

4

1 (iv) $P_P(0,t)$ is a deflator 1.

(v) Deflating 2014 sales to get value totals at 2008 prices yields 1.

Frozen: $\frac{269}{0.986} = 272.82$, Dressed: $\frac{166}{0.973} = 170.61$ 1, 1.

Fingers: $\frac{105}{1.012} = 103.75$ 1.

So $Q_L(2008,2014) = \frac{272.82 + 170.61 + 103.75}{245 + 173 + 111}$ 1.

6 $= \frac{547.18}{529} = 1.03437$ or 103.4% (accept 1dp or better) 1.

(iv) For "Other Seafood" the 2008 price $= \frac{33000}{2750} = 12$ 1+1

$\therefore$ 2014 sales at 2008 prices represent

5 $3130 \times 12 = 37560$ (£) or 37.56 (£K) 1.

$\therefore Q_L^{(2)}(2008,2014) = \frac{547.18 + 37.56}{529 + 33} = 1.040463$ (*) 1.

20 or 104.05% (*) (accept 1.04 – 1.041) 1.

4 (a)(i) $P_L(0,t) = \dfrac{\sum_i q_{oi} p_{oi} \times \frac{p_{ti}}{p_{oi}}}{\sum_i q_{oi} p_{oi}} = \dfrac{\sum_i v_{oi} R_{oti}}{\sum_i v_{oi}}$ 1+1

which is a weighted average of the price relatives, $R_{oti} = p_{ti}/p_{oi}$, with weights given by the values, $v_{oi} = q_{oi} p_{oi}$, at time 0 for commodity i. 1.

1.

4

(ii) Now A has total value $v_{oA} = \sum_{i \in A} v_{oi}$ while B has $v_{oB} = \sum_{i \in B} v_{oi}$.
Similarly the Laspeyres price indices are respectively
$P_{L,A}(0,t) = \dfrac{\sum_{i \in A} v_{oi} R_{oti}}{v_{oA}}$ and $P_{L,B}(0,t) = \dfrac{\sum_{i \in B} v_{oi} R_{oti}}{v_{oB}}$. 1.

$$\therefore P_L(0,t) = \frac{\sum_i v_{oi} R_{oti}}{\sum_i v_{oi}}$$

$$= \frac{\sum_{i \in A} v_{oi} R_{oti} + \sum_{i \in B} v_{oi} R_{oti}}{v_{oA} + v_{oB}} \quad 1.$$

$$= \frac{v_{oA} \sum_{i \in A} \frac{v_{oi}}{v_{oA}} R_{oti} + v_{oB} \sum_{i \in B} \frac{v_{oi}}{v_{oB}} R_{oti}}{v_{oA} + v_{oB}} \quad 1.$$

5

$$= \frac{v_{oA} P_{L,A}(0,t) + v_{oB} P_{L,B}(0,t)}{v_{oA} + v_{oB}} \quad 1.$$

So weights are values, v_{oA} and v_{oB} (as defined.) 1.

(b)(i) $Q_L(2012,2013) = \dfrac{30 \times 100.2 + 50 \times 98.3 + 60 \times 102.9}{30 + 50 + 60}$ 1. 1.

$= 100.6786$ (accept 1dp or better) 1

$Q_L(2013,2014) = \dfrac{31 \times 99.5 + 48 \times 98.5 + 75 \times 105.4}{31 + 48 + 75}$ 1. 1.

6

$= 102.0617$ (accept 1dp or better) 1.

(ii) $Q_L^*(2014;2012) = \dfrac{Q_L(2012,2013) \times Q_L(2013,2014)}{100}$ 1

$= 100.6786 \times 102.0617/100 = 102.75^{(*)}$ 1+1

3

(iii) Chain linking uses more up-to-date (2013) quantity data. 2.

2

(*) accept 102.69 – 102.81

20