

# Adequate sample size for using limiting distributions

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In recent years, non-Bayesian inference of parameters is mostly based on limiting distributions as the sample size, or the number of observational time points in quantitative finance, “goes to infinity” whatever that means. As usual, such limiting theories in statistics do not provide any guideline on the adequate sample size for using the limiting distribution in actual applications, e.g. the blanket usage of normal distribution for Maximum Likelihood Estimator (MLE). Thus the researchers and even the journal referees all seem to act as if this question on sample size should not be asked at all. Thus the applied users happily follow suit in using the limiting distributions without any question on sample size in their own applications. This talk aims to serve a humble reminder with simple examples that the sample size is critical for using a limiting distribution in applications. It is because the convergence to the limiting distribution is NOT a uniform convergence over all the values of the same set of parameters, so that the required sample size can be vastly different being a function of the true parameter value which is not yet known. For the particular limiting normality of MLE, there is another reason adding weight to the requirement of adequate sample size, due to the subtle conflict of the *invariance principle* of limiting normality under a continuous one-to-one transform of MLE (i.e. under any re-parametization) against the *basic principle* of ordinary normality that a random variable (vector) having a normal distribution will not have normal distribution again after any of the foresaid transformations except the linear transform.