EXAMINATIONS OF THE HONG KONG STATISTICAL SOCIETY

HIGHER CERTIFICATE IN STATISTICS, 2012

MODULE 2 : Probability models

Time allowed: One and a half hours

Candidates should answer **THREE** *questions.*

Each question carries 20 marks.
The number of marks allotted for each part-question is shown in brackets.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

The notation log *denotes logarithm to base* ***e***.
Logarithms to any other base are explicitly identified, e.g. $\log_{10}$.

Note also that $\binom{n}{r}$ *is the same as* ${}^nC_r$.

This examination paper consists of 4 printed pages, **each printed on one side only**.
This front cover is page 1.
Question 1 starts on page 2.

There are 4 questions altogether in the paper.

1. The independent random variables X and Y each follow a discrete uniform distribution on the integers 1, 2, 3, 4, 5.

(i) Write down the probability mass function (pmf) $p(x)$ of X. Find $E(X)$ and show that $\text{Var}(X) = 2$.

(7)

(ii) The random variable Z is defined by $Z = XY$.

(a) Write down a list of all the values that Z can take.

(3)

(b) Find the pmf of Z and deduce the mode of Z.

(5)

(c) Find $E(Z)$ and $\text{Var}(Z)$.

(5)

2. (i) The random variable X has a Normal distribution with mean 3 and variance 16. Find $P(X > -2)$.

(4)

(ii) The random variable Y has a Normal distribution with mean 2 and variance 1. X and Y are independent. Find the distribution of $W = X - 3Y$, and find $P(W > 0)$.

(6)

(iii) The independent random variables Y_1, Y_2 and Y_3 have the same distribution as Y, and the random variable V is defined as $X - Y_1 - Y_2 - Y_3$. Find $P(V > 0)$.

(5)

(iv) The independent random variables $X_1, X_2, \ldots, X_n$ have the same distribution as X. Write down in terms of n the distribution of the mean

$$\bar{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$$

and find the least value of n such that $P(\bar{X} > 0) > 0.9995$.

(5)

Turn over

3. A statistics lecturer holds a weekly 'surgery' of 2 hours during which students of his course can visit him to ask for help with their work. It is assumed that individual students arrive randomly and independently during the surgery period (but not before) at a rate of 5 per hour, so that the number of students N_t arriving in any interval of t hours during the surgery period can be taken as having the Poisson distribution

$$P(N_t = n) = \exp(-5t)\frac{(5t)^n}{n!}, \quad n = 0, 1, 2, \ldots .$$

(i) The lecturer makes a cup of coffee immediately before his surgery period begins and will take 10 minutes to drink it. Find the probability that he finishes his coffee before any student arrives.

(3)

(ii) A student has just arrived and asked for help, and his query takes 15 minutes. Find the probability that at least 2 more students arrive while this student is being dealt with.

(4)

(iii) The course runs for a term of 10 weeks, each surgery lasts for 2 hours and surgery visits in different weeks are independent. Use a suitable approximation to calculate the probability that the total number of surgery visits is more than 110. Explain why the exact probability might be anticipated to be more than the calculated value.

(9)

(iv) Suggest reasons why a Poisson model may not be a good assumption in practice.

(4)

4. (i) (a) The continuous random variable X has the exponential distribution with probability density function (pdf) $f(x)$ given by

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0, \lambda > 0.$$

Find the cumulative distribution function (cdf) $F(x)$ of X, and sketch the graph of $F(x)$ for the case $\lambda = \frac{1}{2}$. Mark on your graph the median of X.

(9)

(b) The continuous random variable Y is independent of X and has a distribution with pdf $g(y)$ given by

$$g(y) = \mu e^{-\mu y}, \quad y > 0, \mu > 0.$$

Write down the cdf of Y.

(1)

(ii) Striplights A and B, from two different suppliers, have lifetimes respectively distributed as X and Y in part (i), where $\lambda = \frac{1}{2}$ and $\mu = \frac{1}{3}$, for lifetimes measured in units of 1000 hours. Two new striplights, one from each supplier, are installed at the same time. Their lifetimes may be assumed to be independent.

(a) Find the values of $P(X \leq 2)$ and $P(Y \leq 2)$.

(2)

(b) Find the probability that both striplights last at least 2000 hours, i.e. that $X \geq 2$ and $Y \geq 2$.

(2)

(c) Find the probability that exactly one striplight lasts at least 2000 hours.

(4)

(d) Given that exactly one striplight lasts at least 2000 hours, find the probability that it is A.

(2)